REMARKS

Claims 1-9, 12-14 and 16-19 are pending in this application, with claims 10, 11, 15 and 20 having previously been canceled without prejudice or disclaimer. By the present Amendment, claims 1, 14 and 19 have been amended to clarify the claimed subject matter. Claims 1-9, 12-14 and 16-19 remain pending upon entry of this Amendment, with claims 1, 14 and 19 being in independent form.

Cited Art

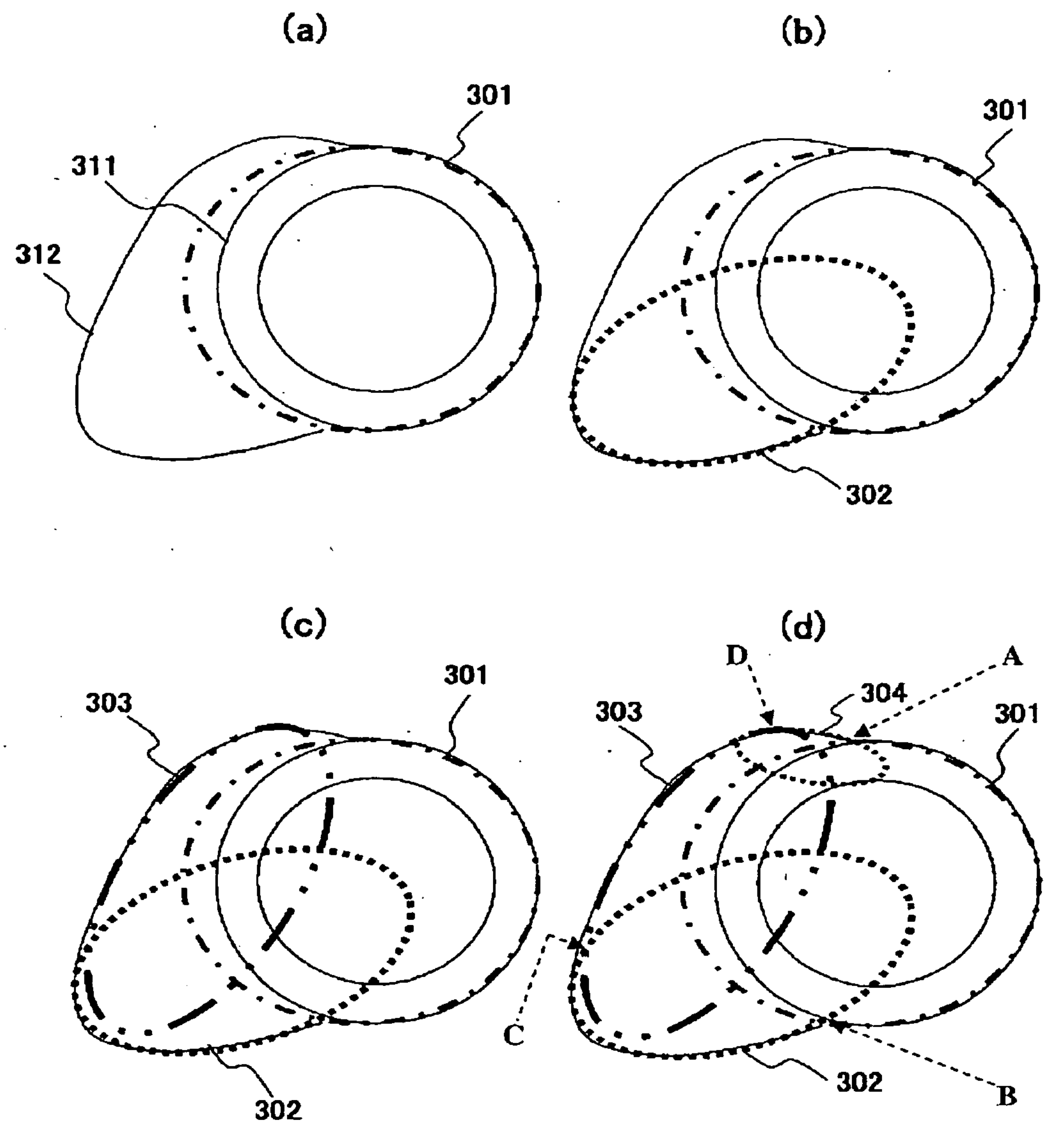
Claim 1-9, 12-14 and 19 were rejected under 35 U.S.C. § 103(a) as purportedly unpatentable over Thomas et al. ("Effect of Black Blood MR Image Quality on Vessel Wall Segmentation") in view of Staib et al., "Parametrically Deformable Contour Models". Claims 16-18 were rejected under 35 U.S.C. § 103(a) as purportedly unpatentable over Thomas in view of Staib and further in view of Barequet et al., "Piecewise-Linear Interpolation between Polygonal Slices".

Applicant respectfully submits that the present application is allowable over the cited art, for at least the reason that the cited art does not disclose or suggest the aspects of the present application of, for each of at least two selected element graphics, approximating at least a ***rectifiable partial contour of the selected element graphics*** to at least a partial contour of a desired region, so that at least two selected element graphics overlap with each other, and making a first closed contour by ***combining at least said rectifiable partial contour of the respective element graphics*** after the approximation.

Such aspects are illustrated by way of example in Fig. 3 [reproduced below, with annotation of Fig. 3(d)] of the present application, and discussed in the application, for example, in paragraph [0068] (of US 2007/0165952 A1). As illustrated in Fig. 3, a closed contour

approximating contour 312 of the right ventricle (desired region) of a heart is obtained by combining partial contours AB, BC, CD, DA of element graphics 301, 302, 303, 304, respectively.

FIG. 3



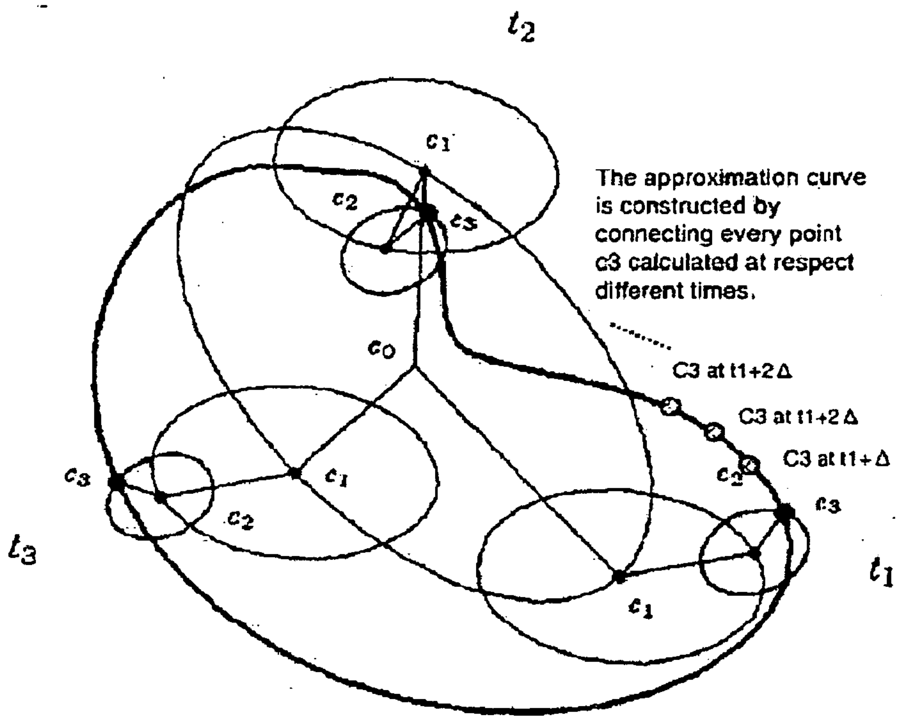
That is, AB corresponds to partial contour of element graphic 301 bounded by point A (of

intersection or overlap as between 301 and 304) and point B (of intersection or overlap as between 301 and 302), BC corresponds to partial contour of element graphic 302 bounded by point B and point C (of intersection or overlap as between 302 and 303), CD corresponds to partial contour of element graphic 303 bounded by point C and point D (of intersection or overlap as between 303 and 304), and DA corresponds to partial contour of element graphic 304 bounded by point D and point A. Partial contours AB, BC, CD, DA of element graphics 301, 302, 303, 304 are clearly well-defined and moreover finite length (and therefore rectifiable, see definition of rectifiable at page 2 of 8 in Exhibit A).

Thomas, as understood by applicant, proposes an approach for approximating arterial wall area through segmenting vessel boundaries. The approach proposed in Thomas requires the operator to draw an approximate outline of vessel of interest (desired region) in the form of a polygon with 4 to 6 vertices, and then each polyline is then deformed towards the inner (lumen/wall) boundary, with additional points automatically introduced as needed to maintain fixed spacing between points. The operator can move the points, if necessary, and then deformation continues.

However, Thomas, as acknowledged in the Office Action, does NOT disclose or suggest, amongst other aspects, approximating at least a ***rectifiable partial contour of the selected element graphics*** to at least a partial contour of a desired region, so that at least two selected element graphics overlap with each other, and making a first closed contour by ***combining at least said rectifiable partial contour of the respective element graphics*** after the approximation..

Staib, as understood by applicant, proposes an approach for segmentation using boundary finding via Fourier decomposition of the boundary, as illustrated in Figure 1 (reproduced below, with annotation), and discussed in Staib, page 99, left column (reproduced below).



3 Parametrization

A generic parametrization for closed contours can be defined based on the elliptic Fourier decomposition of the boundary [6,7]. Any closed contour may be represented by two periodic functions of t, where t varies from 0 to 2a, $x(t)$ and $y(t)$, which can in turn be expressed by their Fourier expansions in matrix form as:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos kt \\ \sin kt \end{bmatrix} \qquad (1)$$

where:

$$\begin{aligned}
a_0 &= \frac{1}{2\pi}\int_0^{2\pi} x(t)\,dt \\
c &= \frac{1}{2\pi}\int_0^{2\pi} y(t)\,dt \\
a_k &= \frac{1}{\pi}\int_0^{2\pi} x(t)\cos kt\,dt \\
b_k &= \frac{1}{\pi}\int_0^{2\pi} x(t)\sin kt\,dt \\
c_k &= \frac{1}{\pi}\int_0^{2\pi} y(t)\cos kt\,dt \\
d_k &= \frac{1}{\pi}\int_0^{2\pi} y(t)\sin kt\,dt
\end{aligned}$$

> This particular version of Fourier boundary representation has a number of advantages. Both the decomposition and the reconstruction can be calculated explicitly and efficiently due to the fast Fourier transform. Since the x and y functions are periodic and continuous, any truncation of the expansion will reconstruct to a closed boundary (unlike some other Fourier representations[8]). The parameters of the decomposition are also more understandable in terms of a geometric interpretation. Invariance to rotation, scale, translation and starting point can also be easily achieved.
>
> In equation 1, the first two coefficients, a_0 and c_0, determine the overall translation of the shape. Each term in the summation can be shown to define an ellipse[6]. The matrix therefore determines the characteristics of the ellipse. The contour can be viewed as being decomposed into a sum of rotating phasors, each individually defining an ellipse, and rotating with a speed proportional to their harmonic number, k. This can be seen in figure 1 where the straight lines represent these phasors for a contour constructed from three ellipses (harmonics). The ellipses, denoted by their center points, c_0, c_1 and c_2 are shown at three different times. The last point, c_3, traces out the curve.

Thus, the closed contour obtained in the approach proposed in Staib is a periodic function of t (time), or stated another way in Staib, the contour is a sum of rotating phasors. In other words and as illustrated by way of annotation in Fig. 1 above, the point c3 traces out the curve over time, and the curve is notionally depicted by connecting the various different c3s over time.

However, such approach of Staib does NOT involve, and Staib simply does NOT disclose or suggest, approximating a rectifiable partial contour of each selected element graphic, nor combination of the rectifiable partial contour of plural element graphics.

It should be noted that the thick black curve traced by point c3 in Figure 1 of Staib intersects with each third ellipse (the third harmonic ellipse) only at one point, but does not tangentially touch to any ellipse, and therefore any rectifiable partial contour of the third ellipses cannot approximate a partial contour of a partial region.

In the present application, a first closed contour is made by combining rectifiable partial contours of respective element graphics after approximating at least the rectifiable partial contour

of the element graphic to at least a partial contour of the partial region. Such closed contour made by combining rectifiable partial contours of the respective element graphics is well-fitted to the entire contour of the desired region as a whole.

Applicant submits that the cited art (including Barequet), even when considered along with common sense and common knowledge to one skilled in the art, does ***NOT*** render unpatentable the above-mentioned aspects of the present application, and that therefore independent claims 1, 14 and 19 are allowable over the cited art.

If the Examiner can suggest an amendment that would advance this application to condition for allowance, the Examiner is respectfully requested to call the undersigned attorney.

If a petition for an extension of time is required to make this response timely, this paper should be considered to be such a petition. The Patent Office is hereby authorized to charge any required fees in connection with this amendment, and to credit any overpayment, to our Deposit Account No. 03-3125.

Examiner's Request for Information under 37 C.F.R. 1.105

Applicant does not have any prior art (other than the information submitted with the Information Disclosure Statements filed on June 14, 2006, June 24, 2008, October 7, 2009 and July 16, 2010) to submit in response to the request.

Respectfully submitted,

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EXHIBIT A

to

AMENDMENT
(U.S. Application No. 10/583,420)

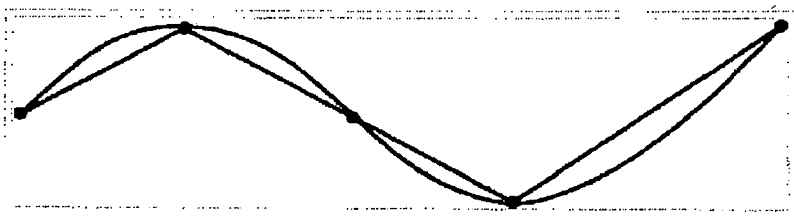
Arc length

From Wikipedia, the free encyclopedia

Determining the **length of an irregular arc segment** is also called rectification of a curve. Historically, many methods were used for specific curves. The advent of infinitesimal calculus led to a general formula that provides closed-form solutions in some cases.

Contents

General approach



Approximation by multiple linear segments

A curve in, say, the plane can be approximated by connecting a finite number of points on the curve using line segments to create a polygonal path. Since it is straightforward to calculate the length of each linear segment (using the Pythagorean theorem in Euclidean space, for example), the total length of the approximation can be found by summing the lengths of each linear segment.

If the curve is not already a polygonal path, better approximations to the curve can be obtained by following the shape of the curve increasingly more closely. The approach is to use an increasingly larger number of segments of smaller lengths. The lengths of the successive approximations do not decrease and will eventually keep increasing—possibly indefinitely, but for smooth curves this will tend to a limit as the lengths of the segments get arbitrarily small.

For some curves there is a smallest number L that is an upper bound on the length of any polygonal approximation. If such a number exists, then the curve is said to be **rectifiable** and the curve is defined to have **arc length** L.

Definition

See also: Lengths of curves

Let C be a curve in Euclidean (or, more generally, a metric) space $X = \mathbf{R}^n$, so C is the image of a continuous function $f\colon [a, b] \to X$ of the interval $[a, b]$ into X.

From a partition $a = t_0 < t_1 < \ldots < t_{n-1} < t_n = b$ of the interval $[a, b]$ we obtain a finite collection of points $f(t_0), f(t_1), \ldots, f(t_{n-1}), f(t_n)$ on the curve C. Denote the distance from $f(t_i)$ to $f(t_{i+1})$ by $d(f(t_i), f(t_{i+1}))$, which is the length of the line segment connecting the two points.

The **arc length** L of C is then defined to be

$$L(C) = \sup_{a=t_0<t_1<\cdots<t_n=b} \sum_{i=0}^{n-1} d(f(t_i), f(t_{i+1}))$$

where the supremum is taken over all possible partitions of $[a, b]$ and n is unbounded.

The arc length L is either finite or infinite. If $L < \infty$ then we say that C is **rectifiable**, and is **non-rectifiable** otherwise. This definition of arc length does not require that C is defined by a differentiable function. In fact in general, the notion of differentiability is not defined on a metric space.

A curve may be parameterized in many ways. Suppose C also has the parameterization $g : [c, d] \to X$. Provided that f and g are injective, there is a continuous monotone function S from $[a, b]$ to $[c, d]$ so that $g(S(t)) = f(t)$ and an inverse function S^{-1} from $[c, d]$ to $[a, b]$. It is clear that any sum of the form $\sum_{i=0}^{n-1} d(f(t_i), f(t_{i+1}))$ can be made equal to a sum of the form $\sum_{i=0}^{n-1} d(g(u_i), g(u_{i+1}))$ by taking $u_i = S(t_i)$, and similarly a sum involving g can be made equal to a sum involving f. So the arc length is an intrinsic property of the curve, meaning that it does not depend on the choice of parameterization.

The definition of arc length for the curve is analogous to the definition of the total variation of a real-valued function.

Finding arc lengths by integrating

See also: Differential geometry of curves

Consider a real function $f(x)$ such that $f(x)$ and $f'(x) = \frac{dy}{dx}$ (its derivative with respect to x) are continuous on $[a, b]$. The length s of the part of the graph of f between $x = a$ and $x = b$ can be found as follows:

Consider an infinitesimal part of the curve ds. According to Pythagoras' theorem $ds^2 = dx^2 + dy^2$, from which:

$$ds^2 = dx^2 + dy^2$$
$$\frac{ds^2}{dx^2} = 1 + \frac{dy^2}{dx^2}$$
$$\frac{ds^2}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^2$$
$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$
$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx.$$

If a curve is defined parametrically by $x = X(t)$ and $y = Y(t)$, then its arc length between $t = a$ and $t = b$ is

$$s = \int_a^b \sqrt{[X'(t)]^2 + [Y'(t)]^2}\,dt.$$

This is more clearly a consequence of the distance formula where instead of a Δx and Δy, we take the limit. A useful mnemonic is

$$s = \lim \sum_a^b \sqrt{\Delta x^2 + \Delta y^2} = \int_a^b \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

If a function is defined in polar coordinates by $r = f(\theta)$ then the arc length is given by

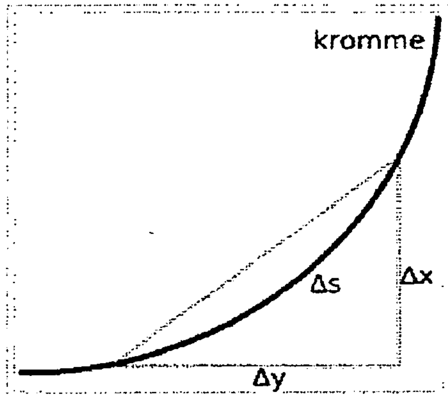
$$s = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta.$$

In most cases, including even simple curves, there are no closed-form solutions of arc length and numerical integration is necessary.

Curves with closed-form solution for arc length include the catenary, circle, cycloid, logarithmic spiral, parabola, semicubical parabola and (mathematically, a curve) straight line. The lack of closed form solution for the arc length of an elliptic arc led to the development of the elliptic integrals.

Derivation

In order to approximate the arc length of the curve, it is split into many linear segments. To make the value exact, and not an approximation, infinitely many linear elements are needed. This means that each element is infinitely small. This fact manifests itself later on when an integral is used.



For a small piece of curve, Δs can be approximated with the Pythagorean theorem

Begin by looking at a representative linear segment (see image) and observe that its length (element of the arc length) will be the differential *ds*. We will call the horizontal element of this distance *dx*, and the vertical element *dy*.

The Pythagorean theorem tells us that

$$ds = \sqrt{dx^2 + dy^2}.$$

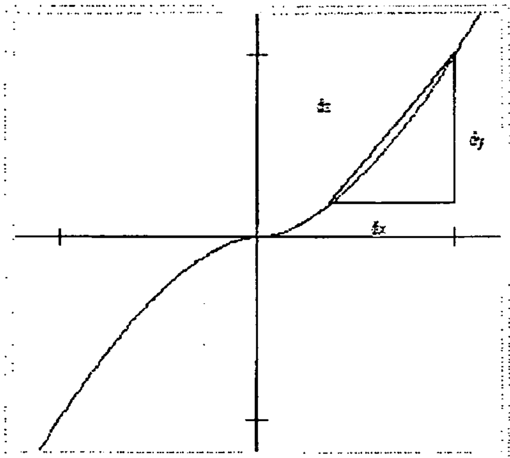
Since the function is defined in time, segments (*ds*) are added up across infinitesimally small intervals of time (*dt*) yielding the integral

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt,$$

If y is a function of x, so that we could take $t = x$, then we have:

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx,$$

which is the arc length from $x = a$ to $x = b$ of the graph of the function f.



A representative linear element of the function $y=t^5, x = t^3$

For example, the curve in this figure is defined by

$$\begin{cases} y = t^5, \\ x = t^3. \end{cases}$$

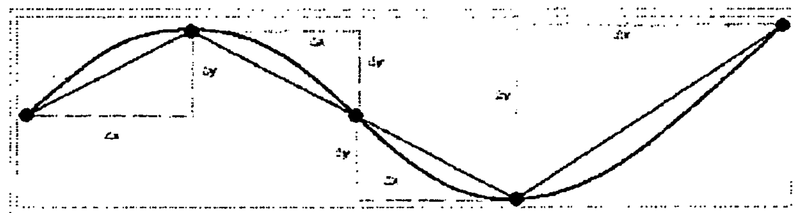
Subsequently, the arc length integral for values of t from -1 to 1 is

$$\int_{-1}^{1} \sqrt{(3t^2)^2 + (5t^4)^2}\, dt = \int_{-1}^{1} \sqrt{9t^4 + 25t^8}\, dt.$$

Using computational approximations, we can obtain a very accurate (but still approximate) arc length of 2.905. An expression in terms of the hypergeometric function can be obtained: it is

$$2\,{}_2F_1\left(-\tfrac{1}{2}, \tfrac{3}{4}; \tfrac{7}{4}; -\tfrac{25}{9}\right).$$

Another way to obtain the integral formula



Approximation by multiple hypotenuses

Suppose that there exists a rectifiable curve given by a function $f(x)$. To approximate the arc length S along f between two points a and b in that curve, construct a series of right triangles whose concatenated hypotenuses "cover" the arc of curve chosen as shown in the figure. For convenience, the bases of all those triangles can be set equal to Δx, so that for each one an associated Δy exists. The length of any given hypotenuse is given by the Pythagorean Theorem:

$$\sqrt{\Delta x^2 + \Delta y^2}$$

The summation of the lengths of the n hypotenuses approximates S:

$$S \sim \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

Multiplying the radicand by $\frac{\Delta x^2}{\Delta x^2}$ produces:

$$\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(\Delta x^2 + \Delta y^2)\frac{\Delta x^2}{\Delta x^2}} = \sqrt{1 + \frac{\Delta y^2}{\Delta x^2}}\Delta x = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\Delta x$$

Then, our previous result becomes:

$$S \sim \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\Delta x_i$$

As the length Δx of these segments decreases, the approximation improves. The limit of the approximation, as Δx goes to zero, is equal to S:

$$S = \lim_{\Delta x_i \to 0} \sum_{i=1}^{\infty} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\Delta x_i = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{1 + [f'(x)]^2}$$

Another Proof (Romil Sirohi's)

We know that the formula for a line integral is $\int_a^b f(x,y) * \sqrt{(x'[t]^2 + y'[t]^2)}dt$. If we set the surface f(x,y) to 1, we will get arc length multiplied by 1, or $\int_a^b \sqrt{(x'[t]^2 + y'[t]^2)}dt$. If x = t, and y = f(t), then y = f(x), from when x is a to when x is b. If we set these equations into our formula we get: $\int_a^b \sqrt{(1 + f'(x)^2)}dx$ (Note: If x = t, dt = dx). This is the arc length formula.

Arcs of circles

The length of an arc of a circle is the central angle divided by 360° multiplied by the circumference.

The circumference of a circle is $C = 2\pi r$, where r is the radius, or $C = \pi d$, where d is the diameter.

In a semicircle, arc length = πr.

Historical methods

Ancient

For much of the history of mathematics, even the greatest thinkers considered it impossible to compute the **length of an irregular arc**. Although Archimedes had pioneered a way of finding the area beneath a curve with his *method of exhaustion*, few believed it was even possible for curves to have definite lengths, as do straight lines. The first ground was broken in this field, as it often has been in calculus, by approximation. People began to inscribe polygons within the curves and compute the length of the sides for a somewhat accurate measurement of the length. By using more segments, and by decreasing the length of each segment, they were able to obtain a more and more accurate approximation. In particular, by inscribing a polygon of many sides in a circle, they were able to find approximate values of π.

1600s

In the 17th century, the method of exhaustion led to the rectification by geometrical methods of several transcendental curves: the logarithmic spiral by Evangelista Torricelli in 1645 (some sources say John Wallis in the 1650s), the cycloid by Christopher Wren in 1658, and the catenary by Gottfried Leibniz in 1691.

In 1659, Wallis credited William Neile's discovery of the first rectification of a nontrivial algebraic curve, the semicubical parabola.

Integral form

Before the full formal development of the calculus, the basis for the modern integral form for arc length was independently discovered by Hendrik van Heuraet and Pierre Fermat.

In 1659 van Heuraet published a construction showing that arc length could be interpreted as the area under a curve—this integral, in effect—and applied it to the parabola. In 1660, Fermat published a more general theory containing the same result in his *De linearum curvarum cum lineis rectis comparatione dissertatio geometrica*.

Building on his previous work with tangents, Fermat used the curve

$$y = x^{3/2}$$

whose tangent at $x = a$ had a slope of

$$\tfrac{3}{2}a^{1/2}$$

so the tangent line would have the equation

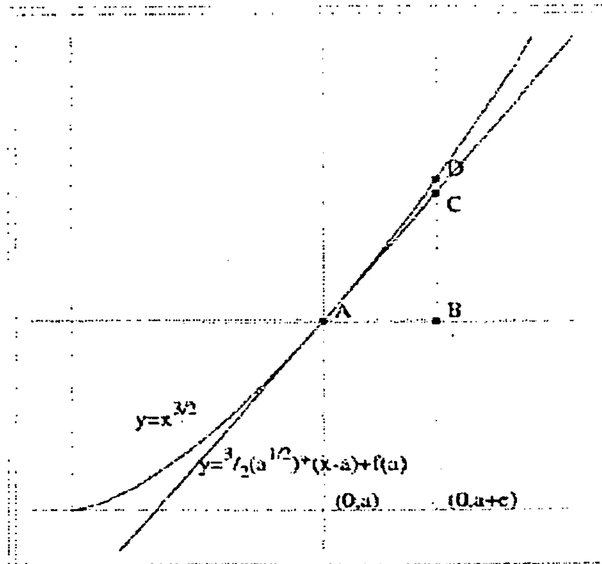
$$y = \tfrac{3}{2}a^{1/2}(x-a) + f(a).$$

Next, he increased a by a small amount to $a + \varepsilon$, making segment AC a relatively good approximation for the length of the curve from A to D. To find the length of the segment AC, he used the Pythagorean theorem:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= \varepsilon^2 + \tfrac{9}{4}a\varepsilon^2 \\ &= \varepsilon^2\left(1 + \tfrac{9}{4}a\right) \end{aligned}$$

which, when solved, yields

$$AC = \varepsilon\sqrt{1 + \tfrac{9}{4}a}\ .$$

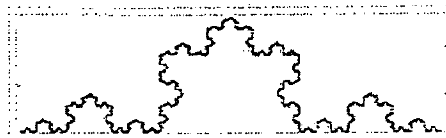


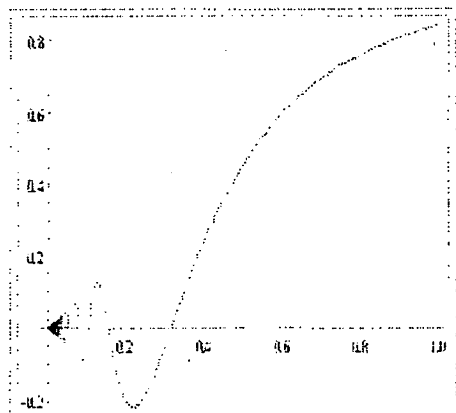
Fermat's method of determining arc length

In order to approximate the length, Fermat would sum up a sequence of short segments.

Curves with infinite length

As mentioned above, some curves are non-rectifiable, that is, there is no upper bound on the lengths of polygonal approximations; the length can be made arbitrarily large. Informally, such curves are said to have infinite length. There are continuous curves on which every arc (other than a single-point arc) has infinite length. An example of such a curve is the Koch curve. Another example of a curve with infinite length is the graph of the function defined by $f(x) = x \sin(1/x)$ for any open set with 0 as one of its delimiters and $f(0) = 0$. Sometimes the Hausdorff dimension and Hausdorff measure are used to "measure" the size of infinite-length curves.


The Koch curve.


The graph of $x\sin(1/x)$.

Generalization to (pseudo-)Riemannian manifolds

Let M be a (pseudo-)Riemannian manifold, $\gamma : [0, 1] \to M$ a curve in M and g the (pseudo-) metric tensor.

The length of γ is defined to be

$$\ell(\gamma) = \int_0^1 \sqrt{\pm g(\gamma'(t), \gamma'(t))}\, dt\ ,$$

where $\gamma'(t) \in T_{\gamma(t)}M$ is the tangent vector of γ at t. The sign in the square root is chosen once for a given curve, to ensure that the square root is a real number. The positive sign is chosen for spacelike curves; in a pseudo-Riemannian manifold, the negative sign may be chosen for timelike curves.

In theory of relativity, arc-length of timelike curves (world lines) is the proper time elapsed along the world line.

See also

- Arc (geometry)
- Integral approximations
- Geodesics
- Meridian arc

References

- Farouki, Rida T. (1999). Curves from motion, motion from curves. In P-J. Laurent, P. Sablonniere, and L. L. Schumaker (Eds.), *Curve and Surface Design: Saint-Malo 1999*, pp. 63–90, Vanderbilt Univ. Press. ISBN 0-8265-1356-5.

External links

- Math Before Calculus (http://math.kennesaw.edu/~jdoto/13.pdf)
- The History of Curvature (http://www3.villanova.edu/maple/misc/history_of_curvature/k.htm)
- Weisstein, Eric W., "Arc Length (http://mathworld.wolfram.com/ArcLength.html) " from MathWorld.
- Arc Length (http://demonstrations.wolfram.com/ArcLength/) by Ed Pegg, Jr., The Wolfram Demonstrations Project, 2007.
- Calculus Study Guide – Arc Length (Rectification) (http://www.pinkmonkey.com/studyguides/subjects/calc/chap8/c0808501.asp)
- Famous Curves Index (http://www-groups.dcs.st-and.ac.uk/~history/Curves/Curves.html) *The MacTutor History of Mathematics archive*
- Arc Length Approximation (http://demonstrations.wolfram.com/ArcLengthApproximation/) by Chad Pierson, Josh Fritz, and Angela Sharp, The Wolfram Demonstrations Project.
- Length of a Curve Experiment (http://numericalmethods.eng.usf.edu/experiments/Length_of_curve_experiment.pdf) Illustrates numerical solution of finding length of a curve.